Olimpiada de matematică
Etapa pe sector
21 februarie 2004

SOLUTII ŞI BAREM DE CORECTARE
Clasa a VII-a

I. a) $\left|3-\sqrt{2}\right|+\left|2\sqrt{2}-3\right|-\left|3\sqrt{2}-5\right|=1$ — 1 p

Finalizare

b) $2\mid n(n+1)\Rightarrow 10\mid 5n(n+1)$ — 2 p

Ultima cifră a lui $5n^2+5n+7$ este 7 — 1 p

$\sqrt{5n^2+5n+7}\notin\mathbb{Q}$ — 1 p

II. a) $S=\dfrac{6}{7}$ — 1 p

b) se analizează 4 cazuri în funcție de semnul lui a şi cel al lui b — 2 p

c) $\left|x-\dfrac{1}{1\cdot 2}\right|+\left|x+\dfrac{1}{2\cdot 3}\right|\geq\left|x-\dfrac{1}{1\cdot 2}-x-\dfrac{1}{2\cdot 3}\right|=\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}$ — 2 p

$$\left|x-\frac{1}{3\cdot 4}\right|+\left|x+\frac{1}{4\cdot 5}\right|\geq\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$$

$$\left|x-\frac{1}{5\cdot 6}\right|+\left|x+\frac{1}{6\cdot 7}\right|\geq\frac{1}{5\cdot 6}+\frac{1}{6\cdot 7}.$$

Prin însumare obținem egalitate. Deci prin punctul b), $x-\dfrac{1}{2}$, $-x-\dfrac{1}{6}$ au acelaşi semn, $x\in\mathbb{Z}$, rezultă $x=0$ care verifică relația deci este soluție. — 2 p

III. a) $\Delta ABC\sim\Delta MNC\Rightarrow\dfrac{AB}{MN}=\dfrac{AC}{MC}=2\Rightarrow AB=2MN$

$m(\sphericalangle MNO)=30^\circ\Rightarrow 2MO=MN\Rightarrow 4MO=AB$ — 3 p

b) $m(\sphericalangle QAM)=m(\sphericalangle AMQ)=m(\sphericalangle NMC)=60^\circ\Rightarrow\Delta AMQ$ echilateral.

Deci $QM=MA=MC\Rightarrow\Delta AQC$ dreptunghic $\Rightarrow CQ\perp AB$ — 4 p

IV. "$\Leftarrow$" $m(\sphericalangle A)=60^\circ\Rightarrow m(\sphericalangle BIC)=120^\circ\Rightarrow m(\sphericalangle B'IC)=m(\sphericalangle DIC)=60^\circ$

Deci $\Delta CID\equiv\Delta CIB'\Rightarrow B'I=ID$, analog $C'I=ID\Rightarrow$

$\Rightarrow\Delta IDC'\equiv\Delta IC'B'\equiv\Delta IB'D\Rightarrow DC'=B'C'=B'D\Rightarrow\Delta B'C'D$ echilateral — 3 p

"$\Rightarrow$" Construim $B'E\perp BC$, $B'F\perp AB\Rightarrow\Delta B'FC'\equiv\Delta B'ED\Rightarrow$

$\Rightarrow\sphericalangle B'C'F\equiv\sphericalangle B'DE$. Analog $\sphericalangle C'BH\equiv\sphericalangle C'DG$, unde $C'G\perp BC$, $C'H\perp AC$ ($E,G\in BC$, $F\in AB$, $H\in AC$).

$m(\sphericalangle B'C'A)+m(\sphericalangle AB'C')=m(\sphericalangle B'DE)+m(\sphericalangle C'DG)=120^\circ\Rightarrow$ — 4 p

$\Rightarrow m(\sphericalangle BAC)=60^\circ$